Optimal Foraging

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Math 102 Section 107

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OSH 3 Recap

Tradeoff between Math and Chemistry depends on difficulty of math exam, k.
https://www.desmos.com/calculator/73d5om1uwo

Recall: from OSH 2

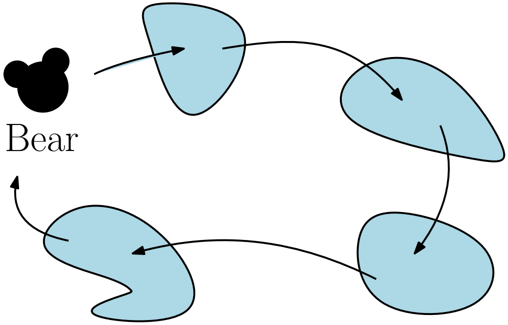
Bears search for berries in patches that grow in patches that can be spread out across a large area. A bear will spend time in one patch gathering food before moving to another patch. The number of berries collected in a patch depends on the amount of time spent in the patch:

$$f(t) = \frac{At}{k + t}$$

https://www.desmos.com/calculator/cwkxvbdpj2

How long should I stay in a food patch?

- Stay too long in one patch - diminishing returns as the patch gets depleted.
- Leave a patch too early - spend unnecessary time traveling.



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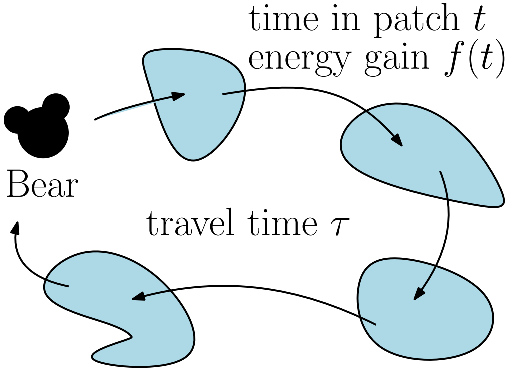

- τ = travel time
- t = time spent in the food patch
- $f(t)$ = energy obtained during time t

Optimal Foraging

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Optimal Foraging

- Nature selects for the organism which can optimize food intake. Two options:
  1. Collect the most food per unit time = average rate of energy gain

  $$R = \frac{\text{energy gained}}{\text{total time spent}}$$

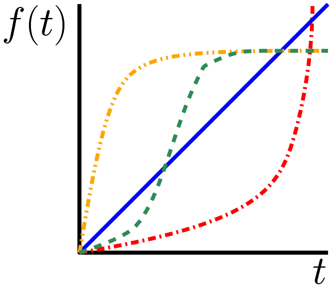
  2. Get the most energy overall = the net energy gain

  $$E = \text{energy in} - \text{energy out}$$

Energy gain $f(t)$

Q1. Which of the following matches the given description of energy gain?

Collection is proportional to the amount of time I spend in the patch.



A. Blue (solid)
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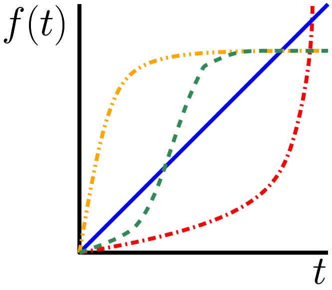

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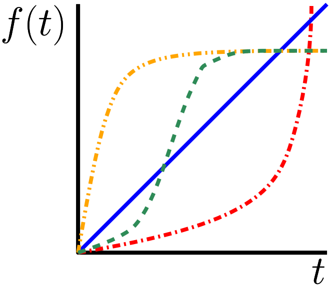
Energy gain $f(t)$

Q2. Which of the following matches the given description of energy gain?

Collection goes well at first but gradually goes down as the resource is depleted.



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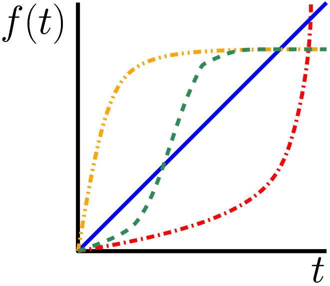

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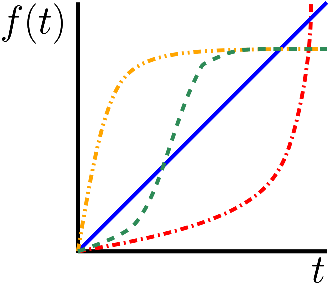
Energy gain $f(t)$

Q3. Which of the following matches the given description of energy gain?

Collection is initially difficult but becomes easier. Eventually there is no more food left.



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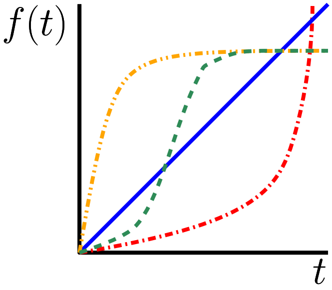

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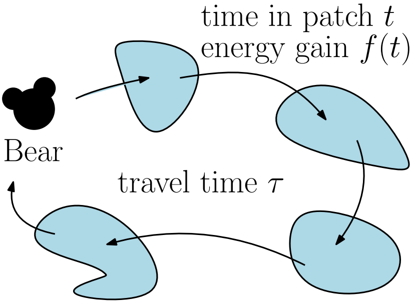
$$R(t) = \frac{f(t)}{t + \tau}.$$

Bear eating berries



$$R(t) = \frac{f(t)}{\tau + t}$$

$$= \frac{At}{(\tau + t)(k + t)}$$

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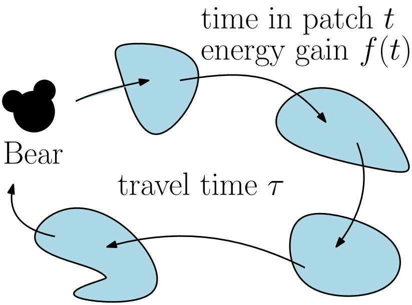

$$R(t) = \frac{f(t)}{\tau + t}$$

$$= \frac{At}{(\tau + t)(k + t)}$$

'If the bear spends t minutes in each patch, the overall average rate of energy gain will be $\frac{At}{(\tau+t)(k+t)}$.'

Sketch $R(t) = \frac{At}{(\tau+t)(k+t)}$

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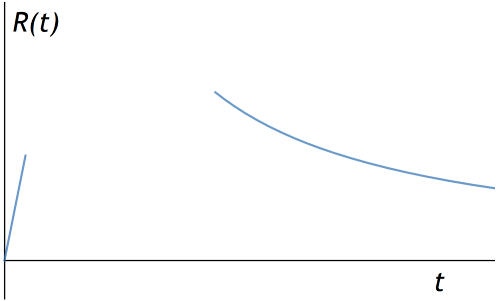

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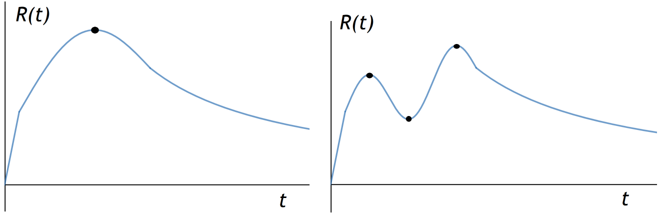

Maximize $R(t) = \frac{At}{(\tau+t)(k+t)}$

- Find the critical points

$$R'(t) = A\frac{k\tau - t^2}{(k+t)^2(\tau+t)^2} = 0$$

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- Keep the positive root: $t = \sqrt{k\tau}$.

Bear eating berries

Are we done? $t = \sqrt{k\tau}$.

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Are we done? $t = \sqrt{k\tau}$. No - we must check that $t = \sqrt{k\tau}$ is a local maximum!

Either check that $R''(t) < 0$ for $t = \sqrt{k\tau}$ OR make a justified argument sketching the function. In this case, because $t = \sqrt{k\tau}$ is the *only* critical point, we can conclude from the sketch that it is the maximum.

Bear eating berries

Q5. Recall that $t = \sqrt{k\tau}$.

If it takes the bear a long time to get to the patch, to maximize the average energy gain, the bear should

A. Stay in the patch for a longer time
B. Stay in the patch less time

Bear eating berries

Q5. Recall that $t = \sqrt{k\tau}$.

If it takes the bear a long time to get to the patch, to maximize the average energy gain, the bear should

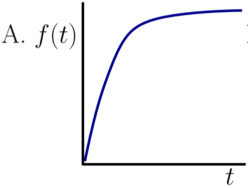
A. Stay in the patch for a longer time

B. Stay in the patch less time

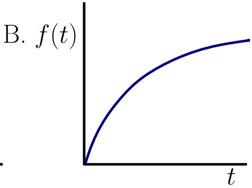
If τ is large, then the optimal time to stay in the patch $t = \sqrt{k\tau}$ is also large.

Bear eating berries

Q6. Recall that $t = \sqrt{k\tau}$.
There are two different patches which take the same amount of time τ to get to. In which patch should the bear spend more time in order to maximize the average energy gain?



$f(t) = \frac{t}{20+t}$



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Bear eating berries

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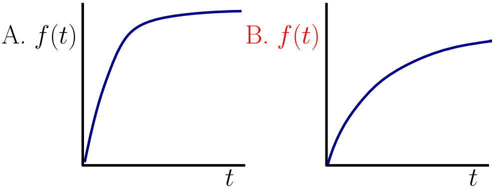

$f(t) = \frac{t}{20+t}$ $\qquad$ $f(t) = \frac{t}{200+t}$

If τ is fixed, but there are two patches, one with k_1 and one with k_2. The bear must stay in patch with the bigger k_i longer, to optimize the average rate of energy gain.

Another application - studying for finals

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Optimal studying:

- As you spend more time studying, you get tired. Your efficiency decreases. Knowledge gained after t hours of study: $f(t)$.
- Length of study break: τ.

Average rate of knowledge gained if you spend t hours before taking a break:

$$R(t) = \frac{f(t)}{\tau + t}$$

The derivative of $R(t) = \frac{f(t)}{\tau+t}$ is

$$R'(t) = \frac{f'(t)(\tau + t) - f(t)}{(\tau + t)^2}$$

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Thus, the critical points t satisfy

$$f'(t) = \frac{f(t)}{\tau + t} = R(t)$$

Marginal Value Theorem

Marginal Value Theorem: To maximize energy intake, the optimal time to stay at a patch of food with energy collection function $f(t)$ with travel time τ is

$$\underbrace{f'(t)}_{\text{instantaneous rate}} = \underbrace{R(t)}_{\text{average rate}}$$

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'Stay at the patch until your gathering rate is slower than the maximum average rate that can be sustained in the long run.'